France
Team Selection Test
2006

Day 1

1. Let $ABCD$ be a square and let Γ be the circumcircle of $ABCD$. M is a point of Γ belonging to the arc CD which doesn't contain A. P and R are respectively the intersection points of (AM) with $[BD]$ and $[CD]$, Q and S are respectively the intersection points of (BM) with $[AC]$ and $[DC]$. Prove that (PS) and (QR) are perpendicular.

2. Let a, b, c be three positive real numbers such that $abc = 1$. Show that:

$$\frac{a}{(a+1)(b+1)} + \frac{b}{(b+1)(c+1)} + \frac{c}{(c+1)(a+1)} \geq \frac{3}{4}.$$

   When is there equality?

3. Let a, b be positive integers such that $b^n + n$ is a multiple of $a^n + n$ for all positive integers n. Prove that $a = b$.

Day 2

1. In a $2 \times n$ array we have positive reals s.t. the sum of the numbers in each of the n columns is 1. Show that we can select a number in each column s.t. the sum of the selected numbers in each row is at most $\frac{n+1}{4}$.

2. It is given a triangle ABC in which $AC + BC = 3AB$. Incircle with center I touches BC and CA at respectively D and E. Let K, L be symmetrical points of D,E with respect to I. Prove that A, B, K, L lie on one circle.

3. Let $M = \{1, 2, \ldots, 3 \cdot n\}$. Partition M into three sets A, B, C which *card* $A =$ *card* $B =$ *card* $C = n$. Prove that there exists a in A, b in B, c in C such that or $a = b + c$, or $b = c + a$, or $c = a + b$

Edited by orl.