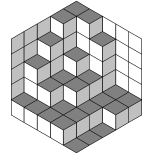




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Day 1

- 1] Let $ABCD$ be a square and let Γ be the circumcircle of $ABCD$. M is a point of Γ belonging to the arc CD which doesn't contain A . P and R are respectively the intersection points of (AM) with $[BD]$ and $[CD]$, Q and S are respectively the intersection points of (BM) with $[AC]$ and $[DC]$. Prove that (PS) and (QR) are perpendicular.
- 2] Let a, b, c be three positive real numbers such that $abc = 1$. Show that:

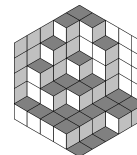
$$\frac{a}{(a+1)(b+1)} + \frac{b}{(b+1)(c+1)} + \frac{c}{(c+1)(a+1)} \geq \frac{3}{4}.$$

When is there equality?

- 3] Let a, b be positive integers such that $b^n + n$ is a multiple of $a^n + n$ for all positive integers n . Prove that $a = b$.



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Day 2

- 1 In a $2 \times n$ array we have positive reals s.t. the sum of the numbers in each of the n columns is 1. Show that we can select a number in each column s.t. the sum of the selected numbers in each row is at most $\frac{n+1}{4}$.
- 2 It is given a triangle ABC in which $AC + BC = 3AB$. Incircle with center I touches BC and CA at respectively D and E . Let K, L be symmetrical points of D, E with respect to I . Prove that A, B, K, L lie on one circle.
- 3 Let $M = \{1, 2, \dots, 3 \cdot n\}$. Partition M into three sets A, B, C which $\text{card } A = \text{card } B = \text{card } C = n$. Prove that there exists a in A, b in B, c in C such that or $a = b + c$, or $b = c + a$, or $c = a + b$

Edited by orl.