Semiring Provenance over Graph Databases

TaPP - King's College London

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Joint work with
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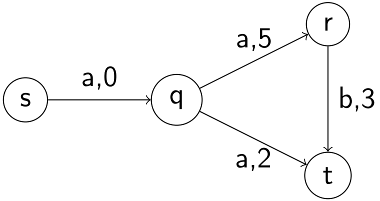
LRI

Contents

Working example



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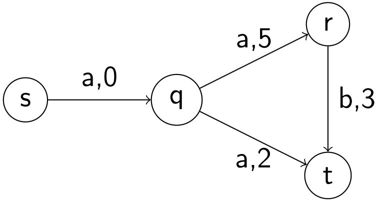

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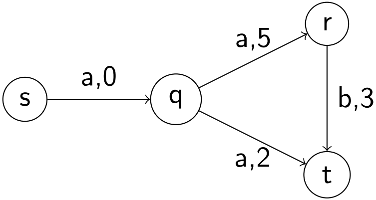

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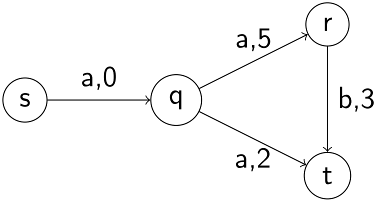

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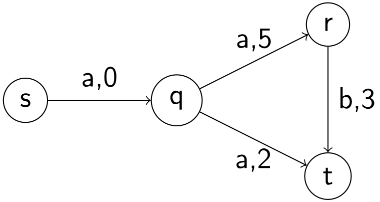

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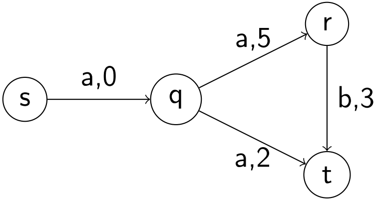

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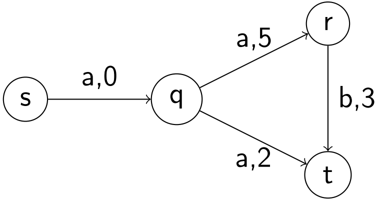

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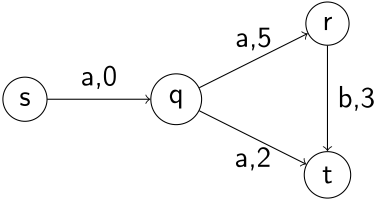

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Overview

We propose several algorithms able to solve:

1. routing information when probabilistic information is present (e.g., road closures, uncertain travel time);
2. top-k relevant paths; or
3. accessibility under security restrictions;

by computing semiring-based provenance of graph queries.

Each of these algorithms yields a tradeoff between generality and time complexity.

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Graph Database with Provenance Indication

Definition (Graph Database)

A *graph database* G over Σ is a pair (V, E), where V is a finite set of node ids and $E \subseteq V \times \Sigma \times V$.

Annotations are given by a weight function, $w : E \to \mathbb{S}$ for $(\mathbb{S}, \oplus, \otimes, \bar{0}, \bar{1})$ a semiring.

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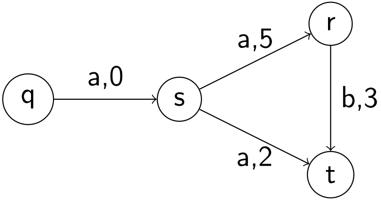

Basic Semiring Theory

An element $a \in \mathbb{K}$ is idempotent if $a \oplus a = a$. $\mathbb{K}$ is said to be idempotent when all elements are.

Definition (k-closed Semiring)

Let $k \geq 0$ be an integer. A semiring $(\mathbb{K}, \oplus, \otimes, \bar{0}, \bar{1})$ is k-closed if $\forall a \in \mathbb{K}, \bigoplus_{n=0}^{k+1} a^n = \bigoplus_{n=0}^{k} a^n$.

Definition (Star semiring)

A star semiring is a semiring $(\mathbb{K}, \oplus, \otimes, \bar{0}, \bar{1})$ with an additional unary operator * verifying:

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Examples of Semirings

Tropical: $\mathbb{T} = (\mathbb{R}_+ \cup \{\infty\}, \mathtt{min}, +, \infty, 0)$ to compute shortest-distance.

kTropical: $\mathbb{T}_k$ is the set of k-tuples in $(\mathbb{R}_+ \cup \{\infty\})^k$ ordered for the natural order of $\mathbb{R} \cup \{\infty\}$):

$$\mathbb{T}_k = \{(a_1, \ldots, a_k) \in (\mathbb{R}_+ \cup \{\infty\})^k : 0 \leq a_1 \leq \cdots \leq a_k\}.$$

$$a \oplus_k b = \mathtt{min}_k((a_i)_i \cup (b_j)_j) \text{ and } a \otimes_k b = \mathtt{min}_k((a_i + b_j)_{i,j})$$

used to compute top-k shortest-distance.

Security: $\mathbb{S}_n$ is then $(\{0, \ldots, n+1\}, \mathtt{min}, \mathtt{max}, n+1, 0)$. Integers correspond to levels such as *Top Secret*, *Secret*, *Restricted*, etc... to compute under security restrictions.

Counting: $(N \cup \{\infty\})$ together with a star operation: $0^* = 1$ and $\forall a \in \mathbb{N}, a^* = \infty$ to compute number of paths between two nodes.

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Querying graph databases with Regular Path Queries (RPQ).

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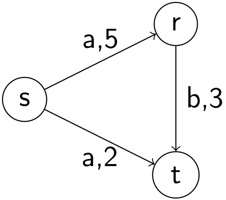

Concept of Provenance

For an answer to a query we want to be able to answer to such questions:

- How is this answer produced?
- What score should this answer receive given initial annotations over edges?

We consider the following problem: given a graph database G with annotations over $\mathbb{K}$, a RPQ Q and s and t the source and target nodes, compute the provenance of the RPQ.

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Can be generalized for star-semirings and is then called Shortest-distance, still not using labels.

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Generalized Floyd-Warshall Algorithm

Partial values we compute are defined as follows

$$l_{ij}^{(k)} := \bigoplus_{\pi \in P_{ij}^{(k)}(G)} w[\pi]$$

Partial values can be computed recursively

$$l_{ij}^{(k)} = l_{ij}^{(k-1)} \oplus \left(l_{ik}^{(k-1)} \otimes (l_{kk}^{(k-1)})^* \otimes l_{kj}^{(k-1)} \right).$$

And the initialisation is

$$l_{ij}^{(0)} = \begin{cases} \lambda(i,j) & \text{if } i \neq j, \\ \bar{1} \oplus \lambda(i,j) & \text{if } i = j. \end{cases}$$

The complexity of this approach is $\mathcal{O}(|V|^3(\mathrm{T}_\oplus + \mathrm{T}_\otimes + T_*))$.
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Generalization from Mohri [1998] of the classical relaxation technique used in Bellman–Ford algorithm.

Modifications to work with non-idempotent semirings. All k-closed semirings.

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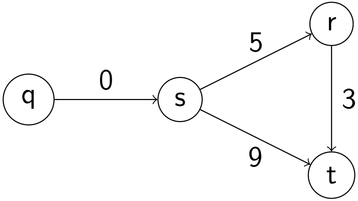

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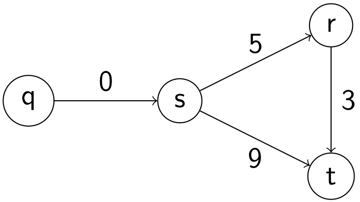

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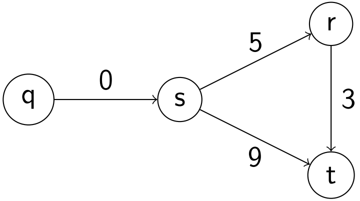

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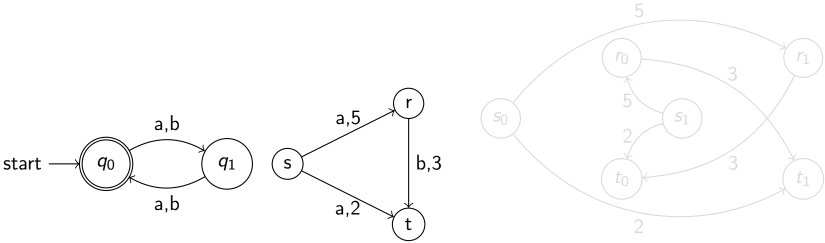

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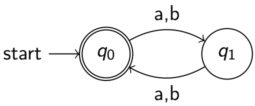

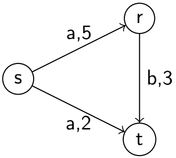

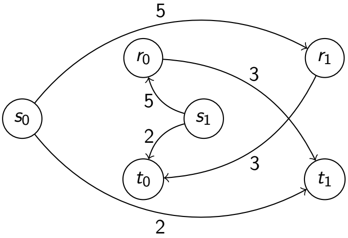

Proof of Correctness

Idea: the label of a valid path leads to an accepting state starting from the initial one.

Theorem

$$\text{prov}^Q_{\mathbb{K}}(G)(x,y) = \bigoplus_{s_F \in F} \text{prov}^R_{\mathbb{K}}(P_{G\times\mathcal{A}_Q})((x,s_0),(y,s_F))$$

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$$\mathrm{prov}_{\mathbb{K}}^{Q}(G)(x,y) = \bigoplus_{s_F \in F} \mathrm{prov}_{\mathbb{K}}^{\mathrm{R}}(P_{G \times \mathcal{A}_Q})((x,s_0),(y,s_F))$$

Applying Mohri's Algorithm

We apply it on the product graph to compute single-source provenance. The semiring needs to be k-closed.

Size of the product graph bounded by $n \cdot |G|$ (n size of the automaton).

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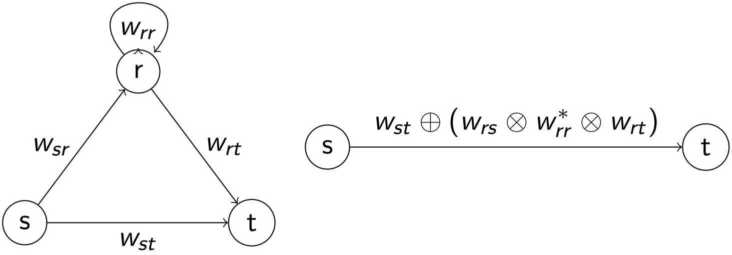

Node Elimination Algorithm (2)

Computing only one time the star for each vertex we can obtain a worst-case complexity in $\mathcal{O}(|V|T_* + |V|^3(T_\oplus + T_\otimes))$.

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Study of Dijkstra Algorithm

Natural order: $a \leq_{\mathbb{K}} b := a \oplus b = a$ is an order when $\oplus$ is idempotent.

0-closedness implies idempotence.

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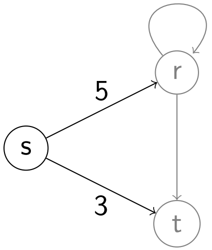

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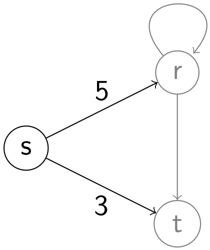

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We used Fibonacci Heap in the implementation, which give a total complexity in $\mathcal{O}(\mathrm{T}_{\oplus}|V|\log|V|) + |E|(T_{\oplus} + T_{\otimes})$.

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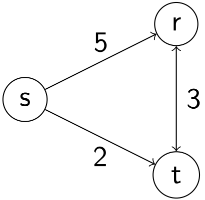

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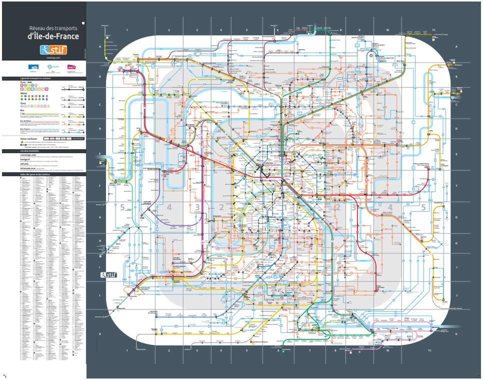

Extraction of the Data

Use tables *stops*, *stop_times*, *trips*, and *routes*.

Labels are lines type and numbers.

Extracted graph contains 16,369 nodes and 41,448 edges.

Subway subgraph contains 302 nodes and 705 edges.

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Graph:	full graph,
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Request:	paths of length mod 2, 3 and 4,
Nodes:	s, t random,

Avg. of 3 runs.

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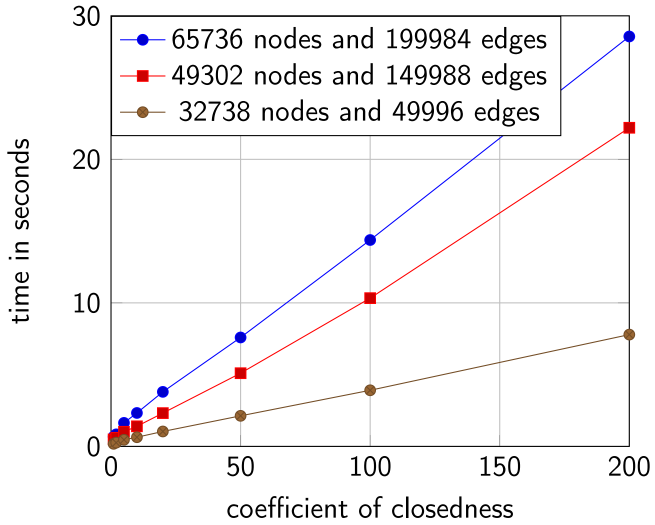

Comparison between Mohri and Node Elimination

Graph:	subway graph,
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Request:	reachability query,
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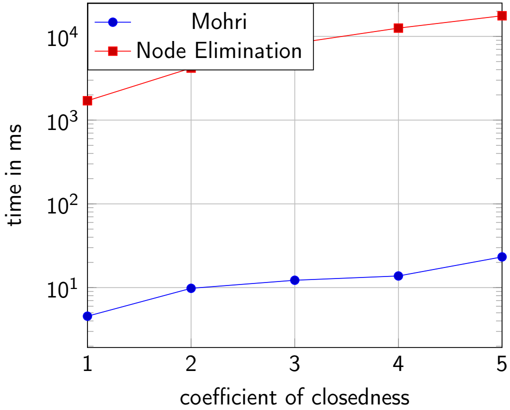

Time (in milliseconds) to compute single-source provenance with Mohri (blue) and Node Elimination (red) depending on the coefficient of closedness.

Comparison between Mohri and Dijkstra

For this purpose we used a graph with random security numbers (0–1000) over edges.

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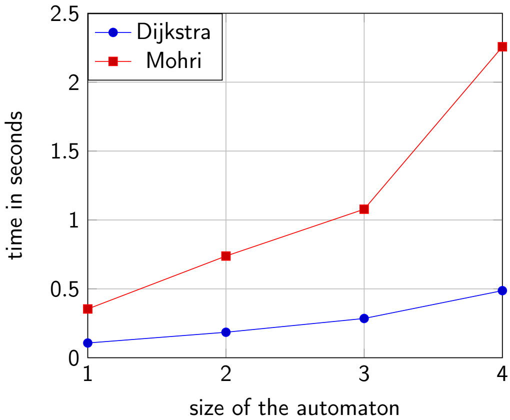

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Perspectives

- Understand practical efficiency of these algorithms.
- Use graph structures and specific properties of transport networks to improve efficiency:
  - contraction hierarchies,
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Bibliography I

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