On the Complexity of Deriving Schema Mappings from Database Instances

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Different sources organize the same data differently

2007		
240	EE	Foto N. Afrati, Chen Li, Jeffrey D. Ullman: Using views to generate efficient evaluation plans for queries. J. Comput. Syst. Sci. 73(5): 703-724 (2007)
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Different sources organize the same data differently

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A Rajaraman, **JD Ullman** - Journal of Computer and System Sciences, 2003 - Elsevier
Several commercial applications, such as online comparison shopping and process automation, require integrating information that is scattered across multiple w ebsites or XML documents. Much research has been devoted to this problem, **...**
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JD Ullman - Proceedings of the 2003 ACM SIGMOD international conference …, 2003 - portal.acm.org
ABSTRACT The education industry has a very poor record of produc- tivity gains. In this brief article, I outline some of the w ays the teaching of a college course in database systems could be made more ecient, and sta time used **...**
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JD Ullman - Database Systems for Advanced Applications, 2003.(DASFAA …, 2003 - ieeexplore.ieee.org
A survey of new directions in database systems. Ullman, JD Stanford University; This paper appears in: Database Systems for Advanced Applications, 2003. (DASFAA 2003). Proceedings. Eighth International **...**
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Motivation

Context

- Multiple data sources containing information about similar entities, with some redundancy (e.g., sources of the deep Web).
- Several different ways to present this information, i.e., several different schemata.
- No a priori information about (some of) these schemata.

How to know the relationships between these schemata, by just looking at the instances?

Other way to see this problem: Match operator on schema mappings, in the setting of data exchange.

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Problem definition

Problem

Given two (relational) database instances I and J with different schemata, what is the optimal description Σ of J knowing I (with Σ a finite set of formulas in some logical language)?

What does optimal implies:

- Conciseness of description.
- Validity of facts predicted by I and Σ.
- All facts of J explained by I and Σ.

(Note the asymmetry between I and J; context of data exchange where J is computed from I and Σ).

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Outline

Source-to-target tuple-generating dependencies

Definition (Source-to-target tgd)

First-order formula of the form:

$$\forall \mathbf{x}\, \varphi(x) \rightarrow \exists \mathbf{y}\, \psi(x, y)$$

with:

- φ conjunction of source relation atoms;
- ψ conjunction of target relation atoms;
- all variables of $\mathbf{x}$ bound in φ.

Example

$$\forall x_1 \forall x_2\ R_1(x_1, x_2) \wedge R_2(x_2) \rightarrow \exists y\ R'(x_1, y)$$

Particular tgds

Two ways of having simpler tgds:

- Disallow existential quantifiers on the right hand-side: full tgds.
- Disallow cycles on both left- and right-hand sides: acyclic tgds. (Classical notion of acyclicity on hypergraphs extending the basic notion of acyclicity on graphs.)

Examples

$\forall x_1 \forall x_2 \forall x_3\ R_1(x_1, x_2) \wedge R_2(x_2, x_3) \wedge R_3(x_3, x_1) \rightarrow R'(x_1)$ is cyclic (and full).

$\forall x_1 \forall x_2 \forall x_3\ R_1(x_1, x_2) \wedge R_2(x_2, x_3) \rightarrow R'(x_1)$ is acyclic (and full).

4 different languages:

$\mathcal{L}_{\text{tgd}}$: arbitrary source-to-target tgds;

$\mathcal{L}_{\text{full}}$: full tgds;

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How to define the pertinence of a set of tgds?

Example

R
a
b
c
d

R'	
a	a
b	b
c	a
d	d
g	h

$$\Sigma_0 = \varnothing$$

$$\Sigma_1 = \{\forall x \ R(x) \rightarrow R'(x, x)\}$$

$$\Sigma_2 = \{\forall x \ R(x) \rightarrow \exists y \ R'(x, y)\}$$

$$\Sigma_3 = \{\forall x_1 \forall x_2 \ R(x_1) \land R(x_2) \rightarrow R'(x_1, x_2)\}$$

$$\Sigma_4 = \{\exists y_1 \exists y_2 \ R'(y_1, y_2)\}$$

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Idea

- Size of a formula: number of occurrences of variables and constants.
- Cost of a schema mapping Σ: Size of the minimum repair of Σ that is valid and explains all facts of J.
- Types of repairs considered:
  - "fix" a universal quantifier by adding conditions ($x = a$ or $x \neq a$);
  - "fix" an existential quantifier by giving corresponding constants ($\tau(\mathbf{x}) \to y = a$ with τ a conjunction of conditions on universally quantified variables);
  - add ground facts to the target instance.
- The problem is then to find a schema mapping of minimal cost.

Example of cost computation

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$$\forall x \; R(x) \to R'(x, x)$$

Predicted R'

a	a
b	b
c	c
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Example of cost computation

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$$\forall x \; R(x) \land x \neq c \rightarrow R'(x, x)$$

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$$\exists y_1 \exists y_2\ R'(y_1, y_2) \land y_1 = c \land y_2 = a$$
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Cost: 17

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Problems considered

Decision problems of interest:

Cost: Is the cost of a given schema mapping less than K?

Optimality: Is a given schema mapping optimal?

Complexity? Algorithms?

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Outline

Behavior for simple operators

Consider the elementary operators of the relational algebra:

- Projection
- Intersection
- Selection (conjunction of atomic conditions)
- Cross Product
- Join (on a given attribute)

Theorem

For any elementary operator γ, the tgd naturally associated with γ is optimal with respect to $(I, \gamma(I))$ (or $(\gamma(J), J)$), under some basic assumptions.

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Examples of naturally associated tgds

Examples

	Condition	I and J	Optimal tgd
Projection	$I \neq \varnothing$ $\pi_1(J) \cap \pi_2(J) = \varnothing$, $\lvert\pi_1(J)\rvert \geqslant 2$	$J = \pi_1(I)$ $I = \pi_1(J)$	$R(x, y) \to R'(x)$ $R(x) \to \exists y\ R'(x, y)$
Selection	$\lvert\sigma_\varphi(I)\rvert \geqslant \frac{\mathsf{size}(\varphi)+2}{3}$ $\sigma_\varphi(J) \neq \varnothing$	$J = \sigma_\varphi(I)$ $I = \sigma_\varphi(J)$	$R(x) \to R'(x)$ $R(x) \to R'(x)$
Product	$R_1^I \neq \varnothing$, $R_2^I \neq \varnothing$ ${R'_1}^J \neq \varnothing$, ${R'_2}^J \neq \varnothing$	$J = R_1^I \times R_2^I$ $I = {R'_1}^J \times {R'_2}^J$	$R_1(x) \wedge R_2(y) \to R'(x, y)$ $R(x, y) \to R'_1(x) \wedge R'_2(y)$

The Polynomial Hierarchy

P	polynomial deterministic algorithm
NP	polynomial non-deterministic algorithm
coNP	complement **NP**
$\boldsymbol{\Sigma_2^P}$	polynomial non-deterministic with Σ_1^P oracle
$\boldsymbol{\Pi_2^P}$	complement $\boldsymbol{\Sigma_2^P}$
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Union of all these classes: **PH** $\subseteq$ **PSPACE**, the polynomial hierarchy.

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(Combined) Complexity Results

	$\mathcal{L}_{\text{tgd}}$	$\mathcal{L}_{\text{full}}$
Cost	Σ_3^P, Π_2^P-hard	Σ_2^P, (co)NP-hard
Optimality	Π_4^P, (co)NP-hard	Π_3^P, (co)NP-hard

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Vertex-Cover in r-partite r-uniform hypergraph

Vertex-Cover: find a set of vertices of minimal size that cover all (hyper)edges in a (hyper)graph.

- NP-complete for general (hyper)graphs.
- PTIME for bipartite graphs (Kőnig's theorem).

Lemma

Vertex-Cover is NP-complete for r-partite r-uniform hypergraphs for $r \geqslant 3$.

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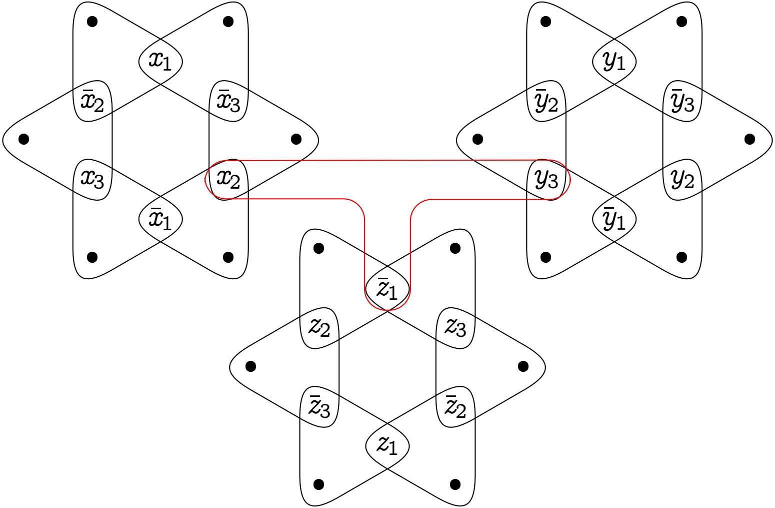
Encoding of 3-SAT



$\neg z \vee x \vee y$

Cost is NP-hard for $\mathcal{L}_{\text{facyc}}$

Reduction from Vertex-Cover in 3-partite 3-uniform hypergraphs.

Without $x = a$ repairs on the left-hand side of a tgd:

- $R(x_1, x_2, x_3) \to R'(x_1)$
- Source instance: hypergraph
- Target instance: empty

Cost: size of the tgd plus twice the minimum size of a vertex cover.

With $x = a$ repairs: a little more difficult, but feasible!

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Outline

Extension to Relational Calculus

- Definition of repairs can be extended to relational calculus.
- Same definition of cost, optimality.
- Cost is not recursive (but co-r.e.).
- Computability of Optimality: open (!).

Other Cost Functions

Why not counting the number of tuples to add or remove in J?

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- Link with Inductive Logic Programming?
- Heuristics?
- Approximation algorithms?
- Generalization of acyclicity?

Merci.