Empirical picture

In general, comparative modified numerals don’t trigger secondary scalar implicatures:

(1) John owns more than three cars.
→ John doesn’t own more than four cars.
But they do when they involve round numerals: (Cummins et al. 2012)

(2) There are more than 90 people in this room.
→ There aren’t more than 100.
As a new observation, they also do when they describe dense quantities:

(3) a. John walked more than 7 kilometers to get home.
→ He didn’t walk 8 kilometers.
b. John has been working there for more than 22 years.
→ He hasn’t been there for 23 years.

Existing theories

- Universal Density of Measurement hypothesis: context-blind mechanism that treats all quantities as dense, and derives a complete absence of secondary implicatures. (Fox and Hackl 2006)
- The UDM has the pattern backwards: we have implicatures precisely when the quantity is dense.
- A more straightforward approach with discrete scales: (Mayr 2013)
  1. “more than four” and “exactly four” are symmetric alternatives;
  2. they cannot be strengthened as they are not innocently excludable;
  3. we predict an ignorance inference.
By itself, this doesn’t account for the difference between (1), (2) and (3).

Building block: granularity scales

Numbers are interpreted within granularity scales (Cummins et al. 2012), based on roundness:

- a: 0, 1, 2, 3...
- b: 0, 10, 20, 30...
- c: 0, 100, 200, 300...
A numeral is usually interpreted within the coarsest scale that it is in.

Formal mechanism: theory #1

Assumptions:
(i) The alternatives of a modified numeral depend on a granularity scale $S$.
(ii) The alternatives of a full sentence are obtained by replacing constituents.
(iii) A more straightforward approach with discrete scales: (Mayr 2013)
  1. “more than four” and “exactly four” are symmetric alternatives;
  2. they cannot be strengthened as they are not innocently excludable;
  3. we predict an ignorance inference.
By itself, this doesn’t account for the difference between (1), (2) and (3).

Handling the examples

- (1) has one equivalent alternative (“John owns four cars.”) and two minimally stronger, symmetric alternatives. We predict an ignorance inference: the speaker isn’t sure whether John has four cars or more.
- The bare numeral alternative to (2) (“There are 100 people here.”) is strictly stronger; we predict a strong implication: there aren’t 100 people here.
- In both cases, we have a disjunctive inference: either the enriched meaning holds, or the alternatives we discuss are irrelevant.
- Plausibly, a total QUD like “how many cars does John own?” makes all alternatives relevant, while a partial QUD like “does John own more than 3 cars?” makes all alternatives irrelevant. Note that the implicatures of (4) disappear under an explicit partial QUD.

Casting doubt on ignorance inferences

- Buccola and Haida (2017) argue that “more than”, in contrast to “at least”, never triggers ignorance inferences.
- Indeed, the predicted inferences from (1) are intuitively dubious.
- Further data shows that we don’t want “more than n” to ever be equivalent to “at least $n + 1$”.

   b. B: More than ??/ / 10 / ??13. (adapted from Buccola and Haida (2017))
(6) We’re in a bar. (Benjamin Spector (p.c.)) Of course she can drink, she’s at least / #more than / 33.

The fixed: structural entailment (theory #2)

The mechanism of Fox and Hackl (2006) relies on context-blind exhaustification: this consists in replacing our contextual entailment relation by structural (or purely logical) entailment. (Magri 2009)

Under structural entailment, whether a sentence entails another doesn’t depend on facts about the world. Thus “John has four cars” is strictly stronger than (1).

- If all alternatives are relevant, exhaustification ought to produce a contradiction: John has more than three but (perhaps) fewer than four cars.
- Hence we predict an obligatory selection of a partial QUD that makes alternatives irrelevant. This is an ignorance inference: how many cars exactly John has is irrelevant, we want to know whether he has more than 3. (This is already noted by Cummins (2013))
- If we don’t want to fall back to the contextual theory, we need to assume that there may be accessible worlds where John answers 9.5 questions, so that the enriched meaning under a total QUD isn’t a contradiction.

Under negation, we predict ignorance inferences, no matter what the quantity of interest is, whether the number is round, and how we define entailment. They turn into inferences about the world under quantifiers:

(10) Nobody here has more than 3 cars / more than 3 hectares of land.
   → Some have as many/much as 3, some have fewer/less.

A look at embedded environments

- Under quantifiers or modals, when talking about discrete objects, we predict ignorance inferences in the structural theory.
- This is usually desirable but sometimes not:
  (7) Every student has brought more than 7 books. → 7 is significant.
  (8) John must own more than 5 cars. → 3 is significant.
  (9) John must answer more than 8 questions to pass. → 9 is the minimal number.
- Contrary to the other examples, (9) behaves like the contextual theory would predict.
- If we don’t want to fall back to the contextual theory, we need to assume that there may be accessible worlds where John answers 9.5 questions, so that the enriched meaning under a total QUD isn’t a contradiction.

Conclusion

- The final mechanism explains how “more than” might trigger implicatures without making it equivalent to “at least”, and explains the difference between round and non-round numerals.
- We claim some reported non-enriched or “ignorant” examples actually trigger ignorance inferences, and provide a system that predicts their patterns.
- We rely on mandatory blind exhaustification, pushing all disambiguation and “pragmatic” phenomena to the choice of QUD.

Bibliography