

Empirical picture

In general, comparative modified numerals don't trigger secondary scalar implicatures:

- (1) John owns more than three cars.
↯ John doesn't own more than four cars.

But they do when they involve round numerals: (Cummins et al. 2012)

- (2) There are more than 90 people in this room.
↔ There aren't more than 100.

As a new observation, they also do when they describe dense quantities:

- (3) a. John walked more than 7 kilometers to get home.
↔ He didn't walk 8 kilometers.
b. John has been working there for more than 22 years.
↔ He hasn't been there for 23 years.

Existing theories

- *Universal Density of Measurement* hypothesis: context-blind mechanism that treats all quantities as dense, and derives a complete absence of secondary implicatures. (Fox and Hackl 2006)
- *The UDM has the pattern backwards: we have implicatures precisely when the quantity is dense.*
- A more straightforward approach with discrete scales: (Mayr 2013)
 1. "more than four" and "exactly four" are symmetric alternatives;
 2. they cannot be strengthened as they are not innocently excludable;
 3. we predict an *ignorance inference*.

By itself, this doesn't account for the difference between (1), (2) and (3).

Building block: granularity scales

Numbers are interpreted within *granularity scales* (Cummins et al. 2012), based on roundness:

- a. 0, 1, 2, 3... b. 0, 10, 20, 30... c. 0, 100, 200, 300...

A numeral is usually interpreted within the coarsest scale that it is in.

Formal mechanism: theory #1

Assumptions:

- (i) The alternatives of a modified numeral depend on a granularity scale S :
(4) $\text{alt}(\text{"more than } n\text{"}) = \{m \text{ (or more)}, \text{"more than } m\text{"}, \text{"exactly } m\text{"} | m \in S\}$
- (ii) The alternatives of a full sentence are obtained by replacing constituents.
- (iii) Any utterance answers a tacit "Question Under Discussion" (QUD) Q which defines a set of "relevant" propositions R_Q .
 1. If ϕ answers Q , for all minimal $\psi \in \text{alt}(\phi) \cap R_Q$ such that ψ asymmetrically entails ϕ , the hearer understands that the speaker doesn't believe that ψ is true (primary implicature). The set of such propositions is $\text{alt}'(\phi; Q)$.
 2. If $\psi \in \text{alt}'(\phi; Q)$ is innocently excludable (Fox 2007), the hearer may infer that the speaker believes that ψ is false (secondary implicature).
 3. Upon hearing ϕ , the hearer infers that the primary and secondary implicatures above hold, and that the propositions in R_Q are irrelevant to the speaker's intention (this is an *irrelevance inference*). If several contextually likely choices for Q lead to pragmatically acceptable meanings, ϕ is ambiguous.

Handling the examples

- (1) has one equivalent alternative ("John owns four cars.") and two minimally stronger, symmetric alternatives. We predict an *ignorance inference*: the speaker isn't sure whether John has four cars or more.
- The bare numeral alternative to (2) ("There are 100 people here.") is strictly stronger; we predict a strong implicature: there aren't 100 people here.
- In both cases, we have a disjunctive inference: either the enriched meaning holds, or the alternatives we discuss are irrelevant.
- Plausibly, a total QUD like "how many cars does John own?" makes all alternatives relevant, while a partial QUD like "does John own more than 3 cars?" makes all alternatives irrelevant. Note that the implicatures of (3) disappear under an explicit partial QUD.

Casting doubt on ignorance inferences

- Buccola and Haida (2017) argue that "more than", in contrast to "at least", never triggers ignorance inferences.
- Indeed, the predicted inferences from (1) are intuitively dubious.
- Further data shows that we don't want "more than n " to ever be equivalent to "at least $n + 1$ ":
(5) A: How many dogs does Ann own? Three? Seven? Twelve? I have no idea.
a. B: At least 5 / 10 / 13.
b. B: More than ??5 / 10 / ??13. (adapted from Buccola and Haida (2017))
- (6) (We're in a bar.) (Benjamin Spector (p.c.))
Of course she can drink, she's (at least / #more than) 33.

The fix: structural entailment (theory #2)

The mechanism of Fox and Hackl (2006) relies on *context-blind exhaustification*: this consists in replacing our contextual entailment relation by *structural* (or *purely logical*) entailment. (Magri 2009)

Under structural entailment, whether a sentence entails another doesn't depend on facts about the world. Thus "John has four cars" is strictly stronger than (1).

- If all alternatives are relevant, exhaustification ought to produce a contradiction: John has more than three but (perhaps) fewer than four cars.
- Hence we predict an obligatory selection of a partial QUD that makes alternatives irrelevant. This is an *irrelevance inference*: how many cars exactly John has is irrelevant, we want to know whether he has more than 3. (this is already noted by Cummins (2013))
- (5) is deviant because the total QUD is introduced very explicitly. (6) is deviant because we know the 18 or 21 threshold is more relevant than the 33 threshold.
- Round numerals and continuous quantities aren't affected by the change.

A look at embedded environments

- Under quantifiers or modals, when talking about discrete objects, we predict irrelevance inferences in the structural theory.
- This is usually desirable but sometimes not:
(7) Every student has brought more than 7 books. ↔ 7 is significant.
(8) John must own more than 3 cars. ↔ 3 is significant.
(9) John must answer more than 8 questions to pass. ↔ 9 is the minimal number.
Contrary to the other examples, (9) behaves like the contextual theory would predict.
- If we don't want to fall back to the contextual theory, we need to assume that there may be accessible worlds where John answers 9.5 questions, so that the enriched meaning under a total QUD isn't a contradiction.
- Under negation, we predict *ignorance inferences*, no matter what the quantity of interest is, whether the number is round, and how we define entailment. They turn into inferences about the world under quantifiers:
(10) Nobody here has more than 3 cars / more than 3 hectares of land.
↔ Some have as many/much as 3, some have fewer/less.

Conclusion

- The final mechanism explains how "more than" might trigger implicatures without making it equivalent to "at least", and explains the difference between round and non-round numerals.
- We claim some reported non-enriched or "ignorant" examples actually trigger irrelevance inferences, and provide a system that predicts their patterns.
- We rely on mandatory blind exhaustification, pushing all disambiguation and "pragmatic" phenomena to the choice of QUD.